Invariant holonomic systems in the bi-Whittaker setting

Wen-Wei Li

Peking University

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Motivation from harmonic analysis

- Let G be a real reductive group, eg. $\mathrm{GL}(n, \mathbb{R})$, and π be an admissible complex representation of G.
- For every $f \in C_c^\infty(G) \otimes$ Haar measure, the operator $\pi(f)$ is known to be of trace-class.
- In fact, one can take f to be a Schwartz function ($=$ rapid decay) on G.

Theorem (“L^1_{loc}”, due to Harish-Chandra, 1964)

The distribution $\Theta_\pi : f \mapsto \mathrm{Tr}(\pi(f))$ on G is represented by a locally L^1 function on G, which is C^ω on $G_{\mathrm{reg,ss}}$.

Corollary (“Determinacy”)

Θ_π is determined by its restriction on any open dense subset.

Later on, Harish-Chandra generalized these to p-adic reductive groups $(\mathrm{char} = 0)$.

Harish-Chandra proved this result by descending to Lie algebras.
Partly motivated by these results, Hotta and Kashiwara (1984) studied the following G-equivariant D-modules on $\mathfrak{g}$

$$\mathcal{M}_\lambda := D(\mathfrak{g}) \big/ \mathrm{relations},$$

called the invariant holonomic systems, where

- $\mathfrak{g}$ is a reductive Lie algebra over $\mathbb{C}$,
- $D(\mathfrak{g})$ is the algebra of algebraic differential operators on $\mathfrak{g}$,
- relations $= \mathrm{ad}\,\mathfrak{g}$-invariance and that $\mathrm{Sym}(\mathfrak{g})^G$ acts via a character λ.

One can define a group version of $\mathcal{M}_\lambda$, replacing $\mathrm{Sym}(\mathfrak{g})^G$ by $\mathcal{Z}(\mathfrak{g}) := Z(\mathcal{U}\mathfrak{g})$.

Among other things, they obtained

Theorem (Hotta–Kashiwara)

$\mathcal{M}_\lambda$ is regular holonomic, equal to the minimal extension of the flat connection $\mathcal{M}_\lambda|_{\mathfrak{g}_{\text{reg.ss}}}$.

This also gives an algebraic proof of Harish-Chandra's theorem that Θ_π is determined by its restriction to any open dense subset.

Further developments

- Sekiguchi (1985) studied invariant holonomic systems on $V := \mathfrak{g}/\mathfrak{h}$ for infinitesimal symmetric pairs $(\mathfrak{g}, \mathfrak{h})$, but the regular holonomicity/minimal extensions remained conjectural.
- Levasseur–Stafford (1999): progress towards Sekiguchi's conjectures
- Laurent (2003) and Galina–Laurent (2004) proved the regular holonomicity for all $(\mathfrak{g}, \mathfrak{h})$, as well as an L^1_{loc} theorem under assumptions on $(\mathfrak{g}, \mathfrak{h})$. Method: *b*-functions.

Recently, Bellamy–Nevins–Stafford (`arXiv:2109.11387`) obtained the

Theorem (Bellamy–Nevins–Stafford)

Let G be a connected reductive $\mathbb{C}$-group. For "robust" infinitesimal symmetric spaces V under G-action, the $D(V)$-module

$$\tilde{\mathcal{G}}_\lambda := D(V) / \left(\mathfrak{g}\text{-invariance} + \mathrm{Sym}(V)^G \text{ acts by } \lambda\right)$$

is regular holonomic and equals to the minimal extension of the flat connection $\tilde{\mathcal{G}}_\lambda|_{V_{\mathrm{reg}}}$.

- V_{reg} is $\{\delta \neq 0\}$ for some discriminant function $\delta \in \mathbb{C}[V]$.
- The assertion about minimal extension amounts to: $\tilde{\mathcal{G}}_\lambda$ has neither submodules nor quotients that are nonzero and supported on $V \smallsetminus V_{\mathrm{reg}}$.

Remarks

- Once lifted to the group level, such results will be helpful for studying *relative characters* in the relative Langlands program.
  Known: relative characters under a pair of *reductive* spherical subgroups $H_1, H_2 \subset G$ generate a regular holonomic D-module (L.. 2022)
- Some results in this direction ("determinacy" of relative characters on the regular locus) are settled by Aizenbud–Gourevitch (2015) and H. Lu (2019) by methods of distributions à la Shalika.
- More generally, one should consider *admissible* modules, i.e. those D-modules with locally finite $\mathrm{Sym}(V)^G$ or $\mathcal{Z}(\mathfrak{g})$-actions.

The bi-Whittaker setting: motivations

Again, we shall motivate from the real setting.

- G: connected reductive $\mathbb{R}$-group,
- $B = TN \subset G$: Borel subgroup,
- $\psi : \mathfrak{n} = \mathrm{Lie}(N) \to \mathbb{C}$ is a non-degenerate character.

Motivation (Baruch, 2001)

Study of *Bessel distributions* on G, which are locally $\mathcal{Z}(\mathfrak{g})$-finite left and right $(\mathfrak{n}, \psi)$-equivariant distributions attached to a pair (λ_1, λ_2) of Whittaker functionals of an admissible representation π of G.

These are one of the earliest examples of "relative characters" alluded to above.

Conjecture

Bessel distributions are L^1_{loc}.

Note: No descent to Lie algebras/Luna slices here, due to nilpotence.

The setting for D-modules

Now consider G, $B = TN$ and ψ over $\mathbb{C}$. Let $\mathrm{W} = \mathrm{W}(G, T)$.

Convention

ℓ = left, r = right.

- Let $N_\ell \times N_\mathrm{r} \subset G_\ell \times G_\mathrm{r}$ act on G.
- Let $\mathfrak{n}_{\ell\mathrm{r}}^{\psi} \subset \mathcal{U}\mathfrak{g}_\ell \otimes \mathcal{U}\mathfrak{g}_\mathrm{r}$ be the Lie subalgebra generated by

$$(\xi - \psi(\xi)) \otimes 1, \quad 1 \otimes (\xi - \psi(\xi)) \quad (\xi \in \mathfrak{n}).$$

- Let $(D(G), \mathfrak{n}_{\ell\mathrm{r}}^{\psi})$-Mod $\subset D(G)$-Mod be the Serre subcategory consisting of modules on which $\mathfrak{n}_{\ell\mathrm{r}}^{\psi}$ acts locally nilpotently — this is what we want to understand.

Fact

There is an explicit equivalence $(D(G), \mathfrak{n}_{\ell\mathrm{r}}^{\psi})$-Mod $\simeq \mathbb{W}$-Mod, where

$$\mathbb{W} := \left(D(G) / D(G)\mathfrak{n}_{\ell\mathrm{r}}^{\psi} \right)^{N_\ell \times N_\mathrm{r}}.$$

- $\mathbb{W}$ is an instance of quantum Hamiltonian reduction:

$$\mathbb{W} = D(G) /\!\!/ \mathfrak{n}_{\ell\mathrm{r}}^{\psi}.$$

- Also known as the quantum Toda lattice, a 2-sided Noetherian ring.
- $\mathcal{Z}(\mathfrak{g}) \hookrightarrow \mathbb{W}$.

Definition (Harish-Chandra modules)

Let $\nu : \mathcal{Z}(\mathfrak{g}) \to \mathbb{C}$ be a character with $\mathfrak{m}_\nu := \ker(\nu)$. The corresponding Harish-Chandra $\mathbb{W}$-module is

$$\mathcal{G}_\nu := \mathbb{W}/\mathbb{W}\mathfrak{m}_\nu.$$

Can check that $\mathcal{G}_\nu$ corresponds to

$$\mathcal{G}_\nu^\natural = D(G)/\left(D(G)\mathfrak{n}_{\ell\mathrm{r}}^{\psi} + D(G)\mathfrak{m}_\nu\right) \in (D(G), \mathfrak{n}_{\ell\mathrm{r}}^{\psi})\text{-Mod}.$$

For representations of infinitesimal character ν, Baruch's Bessel distributions are solutions to $\mathcal{G}_\nu^\natural$.

Definition (admissibility)

We say a $\mathbb{W}$-module is admissible if it is f.g. and $\mathcal{Z}(\mathfrak{g})$ acts locally finitely.

- Admissible $\iff$ generated by finitely many $\mathcal{Z}(\mathfrak{g})$-finite elements;
- Harish-Chandra $\mathbb{W}$-modules are admissible;
- admissible modules have finite length;
- let M be admissible and corresponds to $M^\natural \in (D(G), \mathfrak{n}_{\ell \mathrm{r}}^{\psi})$-Mod, then $M^\natural$ is holonomic, and $j^* M^\natural$ is a flat connection where $j : Bw_0 B \hookrightarrow G$. Proof: apply Aizenbud–Gourevitch–Minchenko 2016 or L. 2022 that

$$\mathrm{SS}(M^\natural) \subset (\mu_\ell \times \mu_\mathrm{r})^{-1} \left(\mathrm{Nilp} \cap (\mathfrak{n}_\ell \times \mathfrak{n}_\mathrm{r})^\perp \right).$$

Remark

The connection $j^* M^\natural$ is almost never regular, due to the exponential behavior along $N_\ell \times N_\mathrm{r}$-orbits.

But the minimal extension $j_{!*} j^* M^\natural$ still makes sense!

Naive questions

Motivated by works by Hotta–Kashiwara, Bellamy–Nevins–Staford et al., we rise the following

Question 1

Let M be an admissible $\mathbb{W}$-module. Is $M^\natural$ equal to $j_{!*}j^* M^\natural$?

This is the same as demanding $M^\natural$ has neither submodules nor quotients that are nonzero and supported on $G \smallsetminus Bw_0 B$. We will formulate it in terms of M.

Question 2

How about the special case $M = \mathcal{G}_\nu$, possibly with conditions on ν?

An affirmative answer would be useful for studying their b-functions, and for the study of Bessel distributions (long term goal).

Kazhdan filtration

Fix an invariant form $(\cdot, \cdot)$ on $\mathfrak{g}$ and a principal $\mathfrak{sl}(2)$-triple $(\mathrm{e}, \mathrm{h}, \mathrm{f})$, such that $\mathrm{e} \in \mathfrak{n}^-$, $\mathrm{f} \in \mathfrak{n}$ and $\psi = (\mathrm{e}, \cdot)$. We are going to define a $\mathbb{Z}$-filtration on $\mathbb{W}$.

- Equip $D(G)$ with the order filtration $D(G)_{\leq \bullet}$.
- The adjoint h-action equips $D(G)$ with a $\mathbb{Z}$-grading. The *Kazhdan filtration* $\mathrm{F}_\bullet D(G)$ is

$$\mathrm{F}_n D(G) = \bigoplus_{d \in \mathbb{Z}} (\mathrm{F}_n D(G))(d), \quad (\mathrm{F}_n D(G))(d) := D(G)_{\leq \frac{n-d}{2}}(d).$$

- Equip $\mathbb{W}$ with the filtration $\mathrm{F}_\bullet \mathbb{W}$ induced by $\mathrm{F}_\bullet D(G)$.

The induced filtration on $\mathcal{Z}(\mathfrak{g})$ is the PBW filtration, i.e. that induced by the PBW filtration on $\mathcal{U}\mathfrak{g}$.

Ginzburg's results: the graded setting

- Let $\mathfrak{c} := \mathfrak{g}^* /\!\!/ G$ and recall that $\mathfrak{g} \simeq \mathfrak{g}^*$ via $(\cdot, \cdot)$.
- Let $\mathfrak{Z} = \mathfrak{Z}_G$ be the *universal centralizer*. It is a smooth symplectic affine variety — to be reviewed.
- The characteristic morphism $\kappa : \mathfrak{Z} \to \mathfrak{c}$ makes $\mathfrak{Z}$ into an affine commutative $\mathfrak{c}$-group scheme; κ is also a Lagrangian fibration.

Theorem (Ginzburg 2018)

The filtration on $\mathbb{W}$ is separating and there is an isomorphism of Poisson algebras

$$\mathrm{gr}(\mathbb{W}) \simeq \mathbb{C}[\mathfrak{Z}]$$

that induces an isomorphism of maximal commutative subalgebras $\mathrm{gr}(\mathcal{Z}(\mathfrak{g})) \simeq \kappa^*(\mathbb{C}[\mathfrak{c}])$.

Ginzburg's result: relation to nil-DAHA

Assume G is simply connected. Take the affine Lie algebra

$$\mathfrak{g}_{\mathrm{aff}} \supset \mathfrak{t}_{\mathrm{aff}}, \quad \tilde{\mathrm{W}} = \mathrm{W} \ltimes X^*(T) \supset \mathrm{W}_{\mathrm{aff}},$$
$$\hbar \in \mathfrak{t}_{\mathrm{aff}} : \text{the minimal imaginary coroot.}$$

Define

$$\mathscr{H}(\mathfrak{t}_{\mathrm{aff}}, \tilde{\mathrm{W}}) := \text{nil-DAHA}$$

generated by $\mathrm{Sym}(\mathfrak{t}_{\mathrm{aff}}) = \mathrm{Sym}(\mathfrak{t})[\hbar]$ and Demazure operators indexed by affine roots. This is a $\mathbb{C}[\hbar]$-algebra.

- There is a $\mathbb{Z}$-grading on $\mathscr{H}(\mathfrak{t}_{\mathrm{aff}}, \tilde{\mathrm{W}})$ such that $\deg \hbar = 1$, and $\hbar$ is not a 0-divisor ($\implies$ a Rees algebra).
- $\mathbb{H} := \mathscr{H}(\mathfrak{t}_{\mathrm{aff}}, \tilde{\mathrm{W}})|_{\hbar=1}$ is thus a $\mathbb{Z}$-filtered algebra with $\mathrm{gr}\mathbb{H} = \mathscr{H}(\mathfrak{t}_{\mathrm{aff}}, \tilde{\mathrm{W}})|_{\hbar=0}$.

The group algebra $\mathbb{C}\mathrm{W}$ embeds naturally in $\mathscr{H}(\mathfrak{t}_{\mathrm{aff}}, \tilde{\mathrm{W}})$. Use the idempotent

$$\mathbf{e} := \frac{1}{|\mathrm{W}|} \sum_{w \in \mathrm{W}} w$$

to define the spherical part $\mathbb{H}^{\mathrm{sph}} := \mathbf{e}\mathbb{H}\mathbf{e}$.

Theorem (Ginzburg 2018)

There is an isomorphism $\mathbb{W} \simeq \mathbb{H}^{\mathrm{sph}}$ of filtered algebras which restricts to $\mathscr{Z}(\mathfrak{g}) \simeq \mathrm{Sym}(\mathfrak{t})^{\mathrm{W}}$ that coincides with Harish-Chandra's isomorphism.

In *loc. cit.*, one defines $\mathscr{H}(\mathfrak{t}_{\mathrm{aff}}, \tilde{\mathrm{W}})^{\mathrm{sph}}$ and the isomorphisms above can be upgraded to the level of Rees algebras.

Proposition

When G is general, $\mathbb{W}$ is 2-sided flat over $\mathscr{Z}(\mathfrak{g})$.

Digression on universal centralizers

There are at least three constructions of $\mathfrak{Z}$.

1. Let $\mathscr{Z}_{\mathrm{reg}}$ be the variety of pairs $(x, g) \in \mathfrak{g}^*_{\mathrm{reg}} \times G$ such that $\mathrm{Ad}^*(g)x = x$. Then $\mathfrak{Z}$ is is the unique affine commutative $\mathfrak{c}$-group scheme whose pullback via $\mathfrak{g}^*_{\mathrm{reg}} \to \mathfrak{c}$ is $\mathscr{Z}_{\mathrm{reg}}$.
   The morphism $\kappa : \mathfrak{Z} \to \mathfrak{c}$ is induced by $(x, g) \mapsto x$.
2. As the bi-Whittaker Hamiltonian reduction $T^*G \;/\!\!/\!\!/ (N_\ell \times N_{\mathrm{r}}, \psi \times \psi)$. Easily described using Kostant slices.
3. As a quotient by $\mathrm{W}(G, T)$ of a certain affine blow-up of $T \times \mathfrak{t}^*$ (Bezrukavnikov–Finkelberg–Mirković 2005). Not directly used here.

The second construction implies $\mathbb{C}[G]^{N_\ell \times N_{\mathrm{r}}} \hookrightarrow \mathbb{C}[\mathfrak{Z}]$. It upgrades to $\mathbb{C}[G]^{N_\ell \times N_{\mathrm{r}}} \hookrightarrow \mathbb{W}$: the filtration on $\mathbb{C}[G]^{N_\ell \times N_{\mathrm{r}}}$ induced from $\mathrm{F}_\bullet \mathbb{W}$ splits!

Study of torsion

$$\mathbb{C}[G]^{N_\ell \times N_\mathrm{r}} = \bigoplus_{\lambda \in X^*(T)_\mathrm{dom}} V_\lambda^{*,N} \otimes V_\lambda^N.$$

Take any $d \in \mathbb{C}[G]^{N_\ell \times N_\mathrm{r}} \smallsetminus \{0\}$ corresponding to regular dominant λ. Then $\{d \neq 0\} = Bw_0B$. Let $\mathcal{S} := d^{\mathbb{Z}_{\geq 0}}$.

Observation

Let M be a $\mathbb{W}$-module and $M^\natural$ the corresponding $D(G)$-module. TFAE:
(a) $M^\natural$ is supported off Bw_0B,
(b) every element of $M^\natural$ is of $\mathcal{S}$-torsion,
(c) every element of M is of $\mathcal{S}$-torsion.

The study of nonzero sub/quot of $M^\natural$ supported off Bw_0B reduces to the study of $\mathcal{S}$-torsion sub/quot of M.

When properly phrased, the notions of good filtrations, singular supports and completions also apply to $\mathbb{Z}$-filtrations.

Let $\mathrm{SS}(M) \subset \mathfrak{Z}$ be the *singular support*/characteristic variety of M, computed using any *good filtration* on M.

Strategy (after Levasseur–Stafford and Bellamy–Nevins–Stafford)

Let Q be a subquotient of an admissible $\mathbb{W}$-module M. When all elements of Q are of $\mathcal{S}$-torsion, we shall look at $\mathrm{SS}(Q)$ w.r.t. any good filtration on Q.

Obstacle

For $\mathbb{Z}$-filtrations, it might happen that $Q \neq 0$ yet $\mathrm{gr}(Q) = 0$, or equivalently the completion $\hat{Q} = 0$.

- This is because $\mathrm{F}_\bullet \mathbb{W}$ is not necessarily a Zariskian filtered ring.
- The problem fades away for modules over the filtered ring $\hat{\mathbb{W}}$: complete $\implies$ Zariskian.

Results modulo completion

All completions are relative to $\mathrm{F}_\bullet \mathbb{W}$ and good filtrations on f.g. $\mathbb{W}$-modules.

Theorem A

Let M be an admissible $\mathbb{W}$-module. If Q is a subquotient of the $\hat{\mathbb{W}}$-module $\hat{M}$ such that Q is generated by $\mathcal{S}$-torsion elements, then $Q = 0$.

- One can show that $\hat{\mathcal{G}}_\nu \neq 0$.
- If we can obtain a "decompleted" version, then $M^\natural \simeq j_{!*} j^* M^\natural$ as desired.
- We are taking completion of modules over non-commutative rings like $\mathbb{W}$, and results such as exactness and Noetherian properties are needed. No problem if one works on the level of Rees algebras (Asensio, van den Bergh, van Oystaeyen 1989).

Below is a sketch of the proof of Theorem A.

1. It suffices to prove $\mathrm{gr}(Q) = 0$ w.r.t. any good filtration, since $\hat{\mathbb{W}}$ is Zariskian as a filtered ring.
2. Admissibility implies $\mathrm{SS}(Q)$ lies in some $\kappa^{-1}(\nu_0)$, which is Lagrangian in $\mathfrak{Z}$. It remains to show $d = 0$ cuts down its dimension ($\mathfrak{Z}$ is symplectic and Gabber's theorem applies to $\mathbb{Z}$-filtered rings as well).
3. Define the big cell $\mathfrak{B}_{w_0} := \{d \neq 0\} \subset \mathfrak{Z}$. We are reduced to show that: every connected component of $\kappa^{-1}(\nu_0)$ intersects $\mathfrak{B}_{w_0}$.
4. Using the compatibility of $\mathfrak{Z}$ with isogenies, we reduce to adjoint G, in which case κ has connected fibers.
5. Claim: for adjoint G, the morphism $\kappa : \mathfrak{B}_{w_0} \to \mathfrak{c}$ is surjective.

The final, hardcore geometric claim is actually in Kostant's 1979 paper: *The Solution to a Generalized Toda Lattice and Representation Theory*. Also re-proved recently by Xin Jin (arXiv:2206.09035).

Homological properties of $\hat{\mathbb{W}}$

Proposition

The ring $\hat{\mathbb{W}}$ is Auslander regular, with self-injective dimension = global dimension = $\dim T$.
Moreover, completions of admissible modules are "holonomic" in the sense that $\mathrm{RHom}_{\hat{\mathbb{W}}}(\cdot, \hat{\mathbb{W}})$ is concentrated at degree $\dim T$, and $\mathrm{Ext}^{\dim T}_{\hat{\mathbb{W}}}(\cdot, \hat{\mathbb{W}})$ gives a duality between such modules (left $\leftrightarrow$ right).

Given the completeness of $\hat{\mathbb{W}}$, the arguments are similar to those for D-modules; the part about duality follows from results of Iwanaga (1997).
The only new input here is

Proposition

For all $\nu \in \mathfrak{c}$ we have $\mathrm{Ext}^m_{\mathbb{W}}(\mathcal{G}_\nu, \mathbb{W}) = 0$ when $m > \dim T$, and when $m = \dim T$ it is the right module version of $\mathcal{G}_\nu$.

Proof. Koszul resolution + flatness of $\mathbb{W}$ over $\mathcal{Z}(\mathfrak{g})$.
For general f.g. M, we have $\mathrm{Ext}^m_{\mathbb{W}}(M, \mathbb{W})^\wedge \simeq \mathrm{Ext}^m_{\hat{\mathbb{W}}}(\hat{M}, \hat{\mathbb{W}})$.

Harish-Chandra modules with general ν

Let $\nu \in \mathfrak{c} \simeq \mathfrak{t}^* /\!\!/ \mathrm{W}$ with preimage $\dot{\nu} \in \mathfrak{t}^*$. Results for $\mathcal{G}_\nu$ without completion are available, provided that ν is $\tilde{\mathrm{W}}$-regular in the sense that no $w \in \tilde{\mathrm{W}} \smallsetminus \{1\}$ stabilizes $\dot{\nu}$.

Theorem B

If $\nu \in \mathfrak{c}$ is $\tilde{\mathrm{W}}$-regular, then $\mathcal{G}_\nu$ has no sub/quot that are nonzero and of $\mathcal{S}$-torsion.

As $\hat{\mathcal{G}}_\nu \neq 0$, it follows from the deeper statement below.

Theorem C

If $\nu \in \mathfrak{c}$ is $\tilde{\mathrm{W}}$-regular, then $\mathcal{G}_\nu$ is a simple $\mathbb{W}$-module.

Idea of proof for Theorem C.

1. Using Ginzburg's result about nil-DAHA (for simply connected G) + ϵ, we deduce
$$D(T)^{\mathrm{W}} \subset \mathbb{W} \subset \mathcal{U}^{-1} D(T)^{\mathrm{W}},$$
where $\mathcal{U}$ is some 2-sided Ore subset, $\mathcal{U} \subset \mathrm{Sym}(\mathfrak{t})^{\mathrm{W}}$ (micro/non-commutative localizations). They are compatible with the embedding of $\mathcal{Z}(\mathfrak{g}) \simeq \mathrm{Sym}(\mathfrak{t})^{\mathrm{W}}$.
2. For $\tilde{\mathrm{W}}$-regular ν, we are reduced to the simplicity of the $D(T)^{\mathrm{W}}$-module $D(T)^{\mathrm{W}} / D(T)^{\mathrm{W}} \mathfrak{m}_\nu$.
3. For each preimage $\dot{\nu} \mapsto \nu$ let $\mathfrak{m}_{\dot{\nu}} \subset \mathrm{Sym}(\mathfrak{t})$ be the maximal ideal. Fact: tensoring by $D(T)$ induces
$$D(T)^{\mathrm{W}}\text{-Mod} \simeq \mathrm{W} \ltimes D(T)\text{-Mod} \simeq D(T)\text{-Mod}^{\mathrm{W\text{-}equi}}.$$
So we only have to show
$$D(T) \otimes_{D(T)^{\mathrm{W}}} (\text{above}) \simeq \bigoplus_{\dot{\nu} \mapsto \nu} D(T)/D(T)\mathfrak{m}_{\dot{\nu}}$$
is simple as a W-equivariant $D(T)$-module. Use arguments akin to Clifford theory.

Questions

1. Can we “decomplete” these results?
2. Alternatively, do the graded parts or completions already contain sufficient information for our purposes? Example: b-functions, etc.
3. Explicit computations in rank 1 case?
4. More general setting, eg. $(N, \psi)\backslash X$ where X is an affine spherical homogeneous G-space?

Thank you for the attention

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